

# The Optimization Model of War and Chess Countermeasure Deployment Based on Mathematical Programming

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**Abstract:** In this paper, first of all, the model assumption and basic rules of the war chess confrontation deployment are set up. On this basis, the standardized model of force deployment is given by using the method of mathematical programming. Then the rationality and feasibility of the model are verified by an example, and the satisfactory results are obtained. This will provide an important basis for the decision of the commander to implement in various situations.

## 1. Introduction

Force is one of the most important implementation resources in the war chess counter deployment. Force deployment is to distinguish, organize and allocate forces according to the implementation objectives of the commander, the rules of the war chess, the enemy forces, the map of the war chess and the performance of the weapons.

The optimization of force deployment is to determine the depth and echelon of force allocation according to the rules of war and chess, the deployment method against the other party and the conditions of war and chess map, so as to meet the task requirements to the maximum. Although it is possible to use simulation method to compare and optimize various plans for the formulation of force deployment plan, such optimization can only be carried out in the Limited plans under consideration, which has great limitations. By using the mathematical programming method, we can get the optimal solution to the simplified deployment problem, and quickly get the optimal solution under the new situation according to the change of enemy forces. If the model is abstracted correctly, the optimal solution can be used as the basis of force deployment directly. Next, we will introduce a military deployment optimization model based on mathematical planning.

## 2. Establishment of Optimization Model of Troops Deployment

Suppose that the two sides of war and chess are red and blue, red is defensive and blue is offensive. The red defense board area is divided into  $i$  defense zones by  $i$  defense lines, and each defense zone is divided into  $j$  defense zones by battle boundaries.

The target of blue attack is to occupy some position of red defense board. According to the judgment of the situation, there may be  $k$  implementation plans for the blue side attack. The blue side's attack and the red side's defense zone are divided into  $i$  stages.

Different implementation plans deploy different forces to different defensive areas of the Red Army in different stages of attack. The task of red defense is to delay the enemy as much as possible, especially to delay the blue side's occupation of the predetermined position.

The red side allocates forces respectively according to  $j$  defensive areas in  $i$  defensive areas, and mobile forces can be deployed to strengthen forces in different defensive areas. The blue advance time of any defense zone is:

$$T = C + KR / B \quad [1] \quad (1)$$

Among them:  $C$  —— non belligerent advance time;  
 $R$  —— total effective force of the red side;  
 $B$  —— total effective force of blue side;

$K$  — proportional constant.

In order to establish a force deployment model [2] using mathematical programming, the following variables are defined:

$T_{jk}$  -- if the blue side adopts the  $k$  scheme, the battle time of at least one position of the red side in the  $j$  sector is lost;

$C_{ijk}$  -- the non belligerent advance time of the blue side in section  $j$  of zone  $i$ ;

$R_{ij}$  - the Red Army's troops invested in the  $j$  section of zone  $i$  at the beginning;

$B_{ijk}$  - the force invested by blue side in phase  $i$ , section  $j$  when implementing scheme  $k$ ;

$R_{ijk}$  - if the blue side adopts the  $k$  scheme for implementation, the red side strengthens its mobile forces in the  $j$  section of zone  $i$ ;

$K_{ij}$  -- in section  $j$  of zone  $i$ , blue advance time / red blue force ratio;

$\gamma_i$  -- the effective ratio of the enhanced forces used in zone  $i$  to the original forces;

$Z_i$  -- the minimum time required by the red side to delay the blue side in zone  $i$ .

### 2.1 Establish Objective Function

In the defense implementation, the purpose of Red Square is to determine the  $R_{ijk}$  value of  $R_{ij}$  and so as to maximize the total combat time  $T$ , even if the minimum value of possible combat time  $T_{jk}$  is the largest, the mathematical expression is:

$$\max T = \min(T_{jk}) \quad (2)$$

### 2.2 Establish Constraints

In zone  $i$ , the delay time shall be  $Z_i$  at least:

$$C_{ij} + K_{ij}(R_{ij} + \gamma_i R_{ijk}) / B_{ijk} \geq Z_i \quad (3)$$

Total number of red forces  $R$ :

$$\sum_{i=1}^i \sum_{j=1}^j R_{ij} + \sum_{i=1}^i \sum_{j=1}^j R_{ijk} = R \quad (4)$$

When the blue side adopts the  $k$  scheme for implementation, the total battle time required for losing at least one position of the red side's  $j$  sector

$$T_{jk} = \sum_{i=1}^i C_{ij} + K_{ij}(R_{ij} + \gamma_i R_{ijk}) / B_{ijk} \quad (5)$$

$R_{ij}$ . Non negative requirements and rounding of  $R_{ijk}$ .

### 2.3 Model Standardization

The whole problem can be transformed into a standard linear programming model by adding several new constraints and a new variable  $t$  [3], that is,  $t \leq T_{jk}$ , and the objective function becomes  $\max T = t$ .

Therefore, the mathematical programming model is established as follows:

$$\max T = t$$

$$ST : C_{ij} + K_{ij} (R_{ij} + \gamma_i R_{ijk}) / B_{ijk} \geq Z_i ;$$

$$\sum_{i=1}^i \sum_{j=1}^j R_{ij} + \sum_{i=1}^i \sum_{j=1}^j R_{ijk} = R ;$$

$$t \leq \sum_{i=1}^i C_{ij} + K_{ij} (R_{ij} + \gamma_i R_{ijk}) / B_{ijk} ;$$

$R_{ij}, R_{ijk}$  is a non negative integer

### 3. Example Application

There are 370 combat units invested by the blue side, which may adopt two plans to carry out the attack. See Table 1 for the forces  $B_{ijk}$  invested by each plan in each defensive zone of the red side.

The red side has 120 combat units for defense. The defense chessboard area is divided into three defense zones by three lines of defense. Each defense zone is divided into two defense zones by the battle boundary, namely  $i = 3, j = 2$ . In defensive zone 1 and zone 2, it is required to delay the blue side's attack for at least 5 days. In defensive zone 2 and zone 3, mobile forces can be mobilized to strengthen their forces, but the effectiveness of the enhanced forces in defensive zone 2 and zone 3 is 0.8 and 0.9 of the original forces, respectively [4].

In section  $j$  of zone  $i$ , the non belligerent advance time of blue side  $C_{ijk}$ ; in section  $j$  of zone  $i$ , the advance time of blue side / the force ratio of red and blue sides  $K_{ij}$  are shown in Table 1.

Table 1 parameter values

$B$	$B_{111} = 40$	$B_{121} = 40$	$B_{211} = 70$	$B_{221} = 50$	$B_{311} = 130$	$B_{321} = 40$
	$B_{112} = 40$	$B_{122} = 40$	$B_{212} = 50$	$B_{222} = 70$	$B_{312} = 90$	$B_{322} = 80$
$C$	$C_{11} = 0.6$	$C_{12} = 1.2$	$C_{21} = 0.8$	$C_{22} = 1.4$	$C_{31} = 0.6$	$C_{32} = 1.6$
$K$	$K_{11} = 10.5$	$K_{12} = 12.6$	$K_{21} = 8.7$	$K_{22} = 13.5$	$K_{31} = 12.4$	$K_{32} = 9.1$

According to the above parameter values and engagement conditions, the model can be established as follows:

$$\max T = t$$

$$ST : 0.6 + 10.5R_{11} / 40 \geq 5;$$

$$1.2 + 12.6R_{12} / 40 \geq 5;$$

$$0.8 + 8.7(R_{21} + 0.8R_{211}) / 70 \geq 5;$$

$$0.8 + 8.7(R_{21} + 0.8R_{212}) / 50 \geq 5;$$

$$1.4 + 13.5(R_{22} + 0.8R_{221}) / 50 \geq 5;$$

$$1.4 + 13.5(R_{22} + 0.8R_{222}) / 70 \geq 5;$$

$$0.6 + 10.5R_{11} / 40 + 0.8 + 8.7(R_{21} + 0.8R_{211}) / 70 + 0.6 + 12.6(R_{31} + 0.9R_{311}) / 130 \geq t;$$

$$0.6 + 10.5R_{11} / 40 + 0.8 + 8.7(R_{21} + 0.8R_{212}) / 50 + 0.6 + 12.6(R_{31} + 0.9R_{312}) / 90 \geq t;$$

$$1.2 + 12.6R_{12} / 40 + 1.4 + 13.5(R_{22} + 0.8R_{221}) / 50 + 1.6 + 9.1(R_{32} + 0.9R_{321}) / 40 \geq t;$$

$$1.2 + 12.6R_{12} / 40 + 1.4 + 13.5(R_{22} + 0.8R_{222}) / 70 + 1.6 + 9.1(R_{32} + 0.9R_{322}) / 80 \geq t;$$

$$R_{11} + R_{12} + R_{21} + R_{22} + R_{31} + R_{32} + R_{211} + R_{221} + R_{311} + R_{321} = 120;$$

$$R_{11} + R_{12} + R_{21} + R_{22} + R_{31} + R_{32} + R_{212} + R_{222} + R_{312} + R_{322} = 120;$$

$R_{ij}, R_{ijk}$  is a non negative integer

The results of solving the linear programming model are as follows: the total delay time is 17.1 days. The force deployment of 120 combat units of the red side is  $R_{11} = 41, R_{12} = 29, R_{21} = 29, R_{22} = 14, R_{211} = 7, R_{222} = 7$ .

This result shows that only two defensive zones and a small amount of mobile forces in the second defensive zone can be deployed by the red side to effectively complete the task of delay.

In order to analyze the influence of the change of situation on the deployment of forces, sensitivity analysis can be carried out<sup>[5]</sup>, and table 2 gives an example of sensitivity analysis results.

Table 2 example of sensitivity result analysis

Conditions	Total delay days	Deployment of the Red Army					
		$R_{11}$	$R_{12}$	$R_{21}$	$R_{22}$	$R_{211}$	$R_{222}$
Original initial conditions	17.1	41	29	29	14	7	7
Red Army increased to 240	34.3	107	84	29	14	6	6
Remove the 5-day limit	20.1	69	51	0	0	0	0
Blue scheme 1 = scheme 2	17.9	44	32	25	19	0	0

As can be seen from table 2:

- (1) When the total number of red troops is doubled, the total lag time is about doubled;
- (2) If there is no requirement for a delay of at least 4 days for each of the two defense zones, the Red Army should be fully deployed in the first defense zone, and no need to retain mobile forces;
- (3) When blue attack plan 1 is equal to plan 2, that is, when the red side clearly understands blue attack plan, the deployment of forces does not need to consider mobile forces.

#### **4. Conclusion**

In this paper, the mathematical programming method is used to establish a model for the simplified deployment of forces, which can be used in many situations. It overcomes the limitation that other methods can only choose the optimal scheme among the limited schemes considered. According to the changes of enemy forces and other parameters, this model can quickly get the optimal solution in the new situation. If the model is abstracted correctly, the optimal solution can be used as the basis of force deployment directly. If the model is too simple, then the optimal solution can at least be used as the basis for further research.

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